UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG 100 1922 2022

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MISG South Africa 2022

Numerical methods for solving singular integral equations with Cauchy-type kernels

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Overview

1. Introduction and background
2. Study Problem
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Introduction and background

Introduction and background
Integral Equations

- Integral equations are equations in which some unknown function to be determined appears under one or several integral signs [5, 6].
- The name integral equation was given by du Bois-Reymond in 1888.
- There are many types of integral equations.
- The classification of integral equations depends mainly on the limits of integration and the kernel of the equation.

Introduction and background

Integral Equations

- Integral equations arise in several fields of science; for example, in elasticity, potential theory, fluid mechanics, biomechanics, approximation theory, plasticity, game theory, queuing theory, medicine, acoustics, heat and mass transfer, economics [4].
- More details about integral equations and their origins can be found in [2, 3].

Introduction and background

Integral Equations

- We will focus our concerned on equations of the form:

$$\fint_{-1}^{1} \frac{\varphi(t)}{t-x}\mathrm{d}t + \int_{-1}^{1} k(x,t)\varphi(t)\mathrm{d}t = f(x), \quad -1 < x < 1, \tag{1}$$

  where the kernel function $k(x,t)$ and the forcing function $f(x)$ are prescribed and the function $\varphi(t)$ is the unknown function to be determined.

- Equation (1) is called Cauchy-type singular integral equation of the first kind and presents a Cauchy-type singularity at $t = x$.

- Singular integral equations with Cauchy kernels appear in many practical problems of elasticity, crack theory, wing theory and fluid flow [1].

Introduction and background

Integral Equations

- The simplest singular integral equation of the first kind has the form

$$\fint_{-1}^{1} \frac{\varphi(t)}{t-x} \mathrm{d}t = f(x), \quad -1 < x < 1. \tag{2}$$

- Equation (2) is called the characteristic singular integral equation and it is obtained when $k(x,t) = 0$ in (1).

- The integral in (2) is understood in the principal value sense and is defined as

$$\fint_{-1}^{1} \frac{\varphi(t)}{t-x} \mathrm{d}t = \lim_{\varepsilon \to 0} \left[\int_{-1}^{x-\varepsilon} \frac{\varphi(t)}{t-x} \mathrm{d}t + \int_{x+\varepsilon}^{1} \frac{\varphi(t)}{t-x} \mathrm{d}t \right], \quad -1 < x < 1. \tag{3}$$

Introduction and background

Integral Equations

- The closed-form solution of the characteristic singular integral equation (2), which is unbounded at both end-points $x = \pm 1$, is given by the formula

$$\varphi(x) = -\frac{1}{\pi^2\sqrt{1-x^2}}\int_{-1}^{1}\frac{\sqrt{1-t^2}f(t)}{t-x}\mathrm{d}t + \frac{C}{\pi\sqrt{1-x^2}}, \tag{4}$$

  where

$$C = \int_{-1}^{1}\varphi(t)\mathrm{d}t. \tag{5}$$

- Equation (2) can also be solved to obtain an approximate analytical solution and a numerical solution.

Study Problem

Study Problem

Cauchy-type singular integral equation

- Consider the problem of solving the singular integral equation given by

$$\frac{1}{\pi}\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_0^1 \frac{\varphi_\xi(\xi)}{\xi - x}\mathrm{d}\xi\right) = 1, \qquad 0 \leqslant x \leqslant 1, \tag{6}$$

  subject to the boundary conditions

$$\varphi_x(0) = 0, \quad \varphi(1) = 0. \tag{7a-b}$$

- We want to solve (6) subject (7a-b) to both analytically and numerically.

Conclusion

QUESTIONS?

References I

J. Cuminato and S. Fitt, A.D.and McKee.
A review of linear and nonlinear cauchy singular integral and integro-differential equations arising in mechanics.
Journal of Integral Equations And Applications, 19:163–207, 2007.

R. Estrada and R. Kanwal.
Singular integral equations.
Springer Science & Business Media, 2012.

J. He.
Some asymptotic methods for strongly nonlinear equations.
International journal of Modern physics B, 20:1141–1199, 2006.

References II

A. Polyanin and A. Manzhirov.
Handbook of integral equations.
2nd ed., CRC Press, Boca Raton, 2008.

A. Wazwaz.
Linear and nonlinear integral equations methods and applications.
Springer Heidelberg Dordrecht London New York, 2011.

A. Wazwaz.
A first course in integral equations.
World Scientific Publishing Company, 2015.